(1) B.Sc. part-I

Indeterminate forms

Def:- If two functions $\phi(x)$ and $\psi(x)$ such that $\phi(a)=0$ [or $\lim_{x\to a}\phi(x)=0$] and $\psi(x)=0$ [or $\lim_{x\to a}\psi(x)=0$, the fraction $\frac{\phi(x)}{\psi(x)}$ is said to assume the indeterminate form $\frac{0}{0}$ at $x=a$ (or as $x\to 0$)

The other simple indeterminate forms are $\frac{\infty}{\infty}$, $0\times\infty$, $\infty-\infty$, 0^0, 1^∞, ∞^0 etc

L'Hopital's Rule for the fundamental form $\frac{0}{0}$

Let $\phi(x)$ and $\psi(x)$ be functions of x such that

(1) $\lim_{x\to a}\phi(x)=0$, $\lim_{x\to a}\psi(x)=0$

and $\phi'(a)$, $\phi'(a)$ and $\psi'(a)\neq 0$

then $\lim_{x\to a}\frac{\phi(x)}{\psi(x)}=\frac{\phi'(a)}{\psi'(a)}$

General form

If $\phi'(a)=\phi''(a)=\cdots=\phi^{n-1}(a)=0$

and $\psi'(a)=\psi''(a)=\cdots=\psi^{n-1}(a)=0$

but $\phi^n(x)$ and $\psi^n(x)$ are not both zero as $x\to a$ then in that case $\lim_{x\to a}\frac{\phi(x)}{\psi(x)}=\lim_{x\to a}\frac{\phi^n(x)}{\psi^n(x)}$

problem ① $\underset{x\to 0}{Lt} \dfrac{xe^x - \log(1+x)}{x^2}$

② Solution:- $\underset{x\to 0}{Lt} \dfrac{xe^x - \log(1+x)}{x^2}$ [form $\frac{0}{0}$]

$= \underset{x\to 0}{Lt} \dfrac{e^x + xe^x - \frac{1}{1+x}}{2x}$ $\left[\frac{0}{0}\right]$

$= \underset{x\to 0}{Lt} \dfrac{e^x + e^x + xe^x + \frac{1}{(1+x)^2}}{2} = \frac{3}{2}$

problem ② $\underset{x\to a}{Lt} \dfrac{\log(x-a)}{\log(e^x - e^a)}$

It is form $\frac{\infty}{\infty}$

$\underset{x\to a}{Lt} \dfrac{\frac{d}{dx}\log(x-a)}{\frac{d}{dx}\log(e^x-e^a)}$

$= Lt \dfrac{\frac{1}{x-a}}{\frac{e^x}{e^x-e^a}} = \underset{x\to a}{Lt} \dfrac{e^x - e^a}{e^x(x-a)}$ $\left[\frac{0}{0}\right]$

$= Lt \dfrac{\frac{d}{dx}(e^x - e^a)}{\frac{d}{dx}(e^x(x-a))}$

$= \underset{x\to a}{Lt} \dfrac{e^x}{e^x(x-a) + e^x} = \dfrac{e^a}{e^a} = 1$

problem ③ $\underset{x\to \frac{\pi}{2}}{Lt} (\sin x)^{\tan x}$

Solution:- Form 1^∞ at $x = \frac{\pi}{2}$

Let $F = \underset{x\to \frac{\pi}{2}}{Lt} (\sin x)^{\cos x}$

then $\log F = \underset{x\to \frac{\pi}{2}}{Lt} \tan x \log \sin x$

$[\infty \times 0]$

(3)

$$= \underset{x \to \frac{\pi}{2}}{Lt} \frac{\log \sin x}{\cot x} \left[\frac{0}{0}\right]$$

$$= \underset{x \to \frac{\pi}{2}}{Lt} \frac{\frac{1}{\sin x} \cdot \cos x}{-\operatorname{cosec}^2 x}$$

$$= \underset{x \to \frac{\pi}{2}}{Lt} (-\sin x \cdot \cos x) = 0$$

Hence $F = e^0 = 1$

Problem (5) $\underset{x \to 0}{Lt} \left(\frac{\tan x}{x}\right)^{\frac{1}{x}}$

Solution:- Let $F = \underset{x \to 0}{Lt} \left(\frac{\tan x}{x}\right)^{\frac{1}{x}} [1^\infty]$

Then $\log F = \underset{x \to 0}{Lt} \frac{\log \tan x - \log x}{x} \left[\frac{0}{0}\right]$

$$= \underset{x \to 0}{Lt} \frac{\frac{\sec^2 x}{\tan x} - \frac{1}{x}}{1} = \underset{x \to 0}{Lt} \left(\frac{1}{\sin x \cos x} - \frac{1}{x}\right)$$

$$= \underset{x \to 0}{Lt} \left(\frac{2}{\sin 2x} - \frac{1}{x}\right) = \underset{x \to 0}{Lt} \frac{2x - \sin 2x}{x \sin 2x} \left[\frac{0}{0}\right]$$

$$= \underset{x \to 0}{Lt} \frac{2 - 2\cos 2x}{2x + 2x\cos 2x} \left[\frac{0}{0}\right]$$

$$= \underset{x \to 0}{Lt} \frac{4\sin 2x}{2\cos 2x + 2\cos 2x - 4x\sin 2x}$$

$= 0$

Hence $F = e^0 = 1$

problem (5) $\underset{x\to 0}{Lt}\left(\frac{\sin x}{x}\right)^{x^{1/2}}$

soln:- let $F = \underset{x\to 0}{Lt}\left(\frac{\sin x}{x}\right)^{\frac{1}{x^2}}$

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Then $\log F = \underset{x\to 0}{Lt}\ \frac{\log \sin x - \log x}{x^2}\left[\frac{0}{0}\right]$

$= \underset{x\to 0}{Lt}\ \frac{\frac{\cos x}{\sin x} - \frac{1}{x}}{2x}$

$= \underset{x\to 0}{Lt}\ \frac{x\cos x - \sin x}{2x^2 \sin x}\left[\frac{0}{0}\right]$

$= \underset{x\to 0}{Lt}\ \frac{\cos x - x\sin x - \cos x}{4x\sin x + 2x^2\cos x}$

$= \underset{x\to 0}{Lt}\ \frac{-\sin x}{4\sin x + 2x\cos x}\left[\frac{0}{0}\right]$

$= \underset{x\to 0}{Lt}\ \frac{-\cos x}{4\cos x + 2\cos x - 2x\sin x} = -\frac{1}{6}$

$-1/6$

Hence $F = e^{-1/6}$